Sec. 9.3 Trigonometric Models

Ex: A utility company serves two different cities. Let P_1 be the power requirement in megawatts for City 1 and P_2 be the requirement for City 2. Both P_1 and P_2 are functions of t, the number of hours elapsed since midnight. Suppose P_1 and P_2 are given by the following formulas:

$$P_1 = 40 - 15\cos\left(\frac{\pi}{12}t\right) \quad \text{and} \quad P_2 = 50 + 10\sin\left(\frac{\pi}{12}t\right).$$

$A = 10$

$P = \frac{2\pi}{\frac{\pi}{12}} = 2\pi \cdot \frac{12}{\pi} = 24$

midline: $y = 50$

a. Describe the power requirements of each city in words.

P_1: midline: $y = 40$ – reflected cosine – $P = \frac{2\pi}{\frac{\pi}{12}} = 2\pi \cdot \frac{12}{\pi} = 24$

P_1 starts with a 25 mw requirement at midnight, at 6 AM it is back to its average (40), at noon, it's at the maximum of 55, at 6PM it is down to 40. It will return to the low of 25 at midnight.

P_2 starts at its average of 50 at midnight, rises to 60 at 6AM, back to 50 at noon, down to 40 at 6PM and back to 50 at midnight.

b. What is the maximum total power the utility company must be prepared to provide?

$P_1 + P_2 = 40 - 15\cos\left(\frac{\pi}{12}t\right) + 50 + 10\sin\left(\frac{\pi}{12}t\right)$

$= 90 + 10\sin\left(\frac{\pi}{12}t\right) - 15\cos\left(\frac{\pi}{12}t\right)$

$= 90 + A\sin\left(\frac{\pi}{12} + \phi\right)$

$P_1 + P_2 = \boxed{90 + \sqrt{325}\sin\left(\frac{\pi}{12} - .9823\right)}$

$a_1 = 10$
$a_2 = -15$
$A = \sqrt{10^2 + (-15)^2}$
$= \sqrt{325}$
≈ 18.028

$\phi = \tan^{-1}\frac{a_2}{a_1} = -\frac{15}{10}$

$\phi = \tan^{-1}\left(-\frac{3}{2}\right)$

$\phi = -.9823$

$\cos\phi = \frac{a_1}{A}$
$= \frac{10}{\sqrt{2725}}$

$\sin\phi = \frac{a_2}{A}$
$= \frac{-15}{\sqrt{2725}}$

Q4

MAX WHEN SINE = 1 $\Rightarrow 90 + \sqrt{325}(1)$

$\boxed{= 108.028 \text{ megawatts}}$

Ex: Sketch and describe the graph of $y = \sin 2x + \sin 3x$.

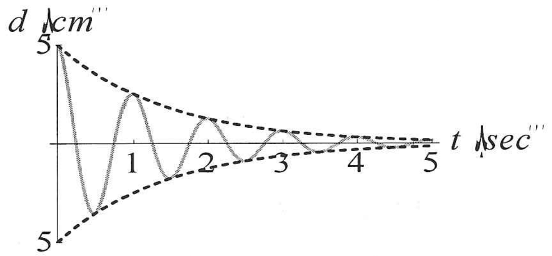
SEE GRAPH ON 9.3 PART 2

Damped Oscillation (Read the example on pages 387-389.)

If A_0, B, and C, and k are constants, $k > 0$, a function of the form

$$y = A_0 e^{-kt} \cos(Bt) + C \text{ or } y = A_0 e^{-kt} \sin(Bt) + C$$

can be used to model an oscillating quantity whose amplitude decreases exponentially according to $A(t) = A_0 e^{-kt}$ where A_0 is the initial amplitude. Our model for the displacement of a weight is in this form with $k = \ln 2$.

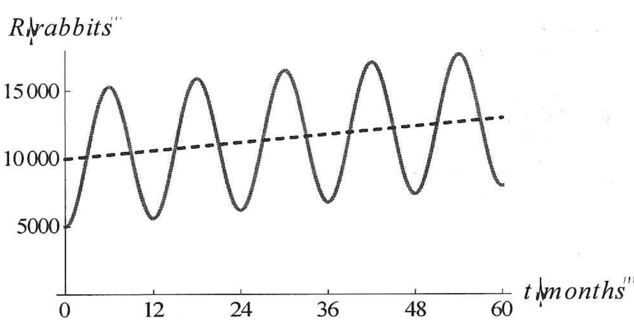


A graph of the weight's displacement assuming that the amplitude is a decreasing exponential function of time

Oscillation With A Rising Midline We represented a rabbit population undergoing seasonal fluctuations by the function

$$R = f(t) = 10000 - 5000 \cos(\pi/6t),$$

where R is the size of the rabbit population t months after January. Now let us imagine a different situation. What if the average, even over long periods of time, does not remain constant? For example, suppose that, due to conservation efforts, there is a steady increase of 50 rabbits per month in the average rabbit population. Thus, we could write an equation where 10000 is the average value (constant midline) and the – 5000 cos (π/6 t) is the seasonal variation. The additional 50t is the midline population increasing by 50 every month.



$R = 10000 + 50t$

$R = 10{,}000 + 50t - 5000 \cos(\pi/6\ t)$

Ex: Look at and read the Acoustic Beats section on pages 390-391. Discuss (with your partners) how the solution was arrived at and your level of understanding. Can you interpret the graph?

HW: pg 392-394 #3-6